$$\frac{2x}{x^2-1} < \frac{1}{x^2-1}$$

$x^2-1<0$

$$\frac{2x}{x^2-1} > \frac{1}{x^2-1}$$

$\cdot 2$

$\cancel{x}\ > \frac{1}{2}$

$x^2-1>0$:

$x^2-1>0 \wedge x<\frac{1}{2}$

$x \in (-\infty; -1) \cup (\frac{1}{2}; 1)$

2 a)

$\forall a, b \in \mathbb{R}$

$|a+b| \leq |a|+|b|$

$|x| = \max\{x; -x\}$

$a \leq |a| \qquad b \leq |b| \qquad a+b \leq |a|+|b|$

$-a \leq |a| \qquad -b \leq |b| \qquad -a-b = |a|+|b|$

$|a+b| = a+b \vee |a+b| = -a-b$

$|a+b| \leq |a|+|b|$

2 b) $||a|-|b|| \leq |a-b|$

$|a+b-b| \geq \pm a$

$|a-b| \geq |a|-|b|$

$|a-b|+|b| \geq |a+b-b|$

$|b-a| \geq |b|-|a|$

$|a-b|+|b| \geq \pm a$

$|a-b|+|b| \geq |a|$

3 a) $\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}: \ y > x$

$x \in \mathbb{N}: \ y = x + 1$

NEGACE

$\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}: \ y \leq x$

3 b) $\forall x \in \mathbb{N}\ \exists \cancel{x} y \in \mathbb{N}: \ x < y$

$x = 1$

NEGACE

$\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}: \ x \geq y$

3 f) $\forall a \in \mathbb{R}\ \exists \varepsilon > 0\ \exists \alpha \in \mathbb{R}\ \forall x \in \mathbb{R}:$

$x \in (a, a+\varepsilon) \Leftrightarrow |x - \alpha| < 1$

NEGACE

$\exists a \in \mathbb{R}\ \forall \varepsilon > 0\ \forall \alpha \in \mathbb{R}\ \exists x \in \mathbb{R}:$

$[x \notin (a, a+\varepsilon) \wedge |x - \alpha| < 1] \vee [x \in (a, a+\varepsilon) \wedge |x - \alpha| >$

$\left|x - \left(a + \frac{\varepsilon}{2}\right)\right| < \frac{\varepsilon}{2} \Leftrightarrow |x - \alpha| < 1$

$x \in (a, a+\varepsilon) \Leftrightarrow x \in (\alpha - 1, a + 1)$

$\varepsilon = 2:$

$\alpha = a + \frac{\varepsilon}{2}$

$\alpha = a + 1$

1. A)

$x^2 - 4x - 5 > 0$

$D = b^2 - 4ac$

$D = 16 - 1\cdot(-5)$

$D = 25$

$x_{1,2} = \frac{-b^2 \pm \sqrt{D}}{2} = \frac{-4 \pm 5}{2}$

$= -\frac{9}{2}$, $\frac{1}{2}$

$\left(x + \frac{9}{2}\right)\left(x + \frac{1}{2}\right)$

1. B)

$-1 \quad 1$

$x \in (-\infty; 1)$: $-x + 1 + x - 1 = 1$

$-2 = 1$

$x \in (-1; 1)$: $x + 1 + x + 1 = 1$

$2x = 1$

$\boxed{x = \frac{1}{2}}$

$\frac{1}{2} \in (-1; 1)$

$x \in (1; \infty)$: $x + 1 - x + 1 = 1$

$2 = 1$

---

1. c) $\frac{x}{x-1} - \frac{x}{x+1} < \frac{1}{x^2 - 1}$ $\quad x \neq 1, -1$

$\frac{x}{x-1} - \frac{x}{x+1} < \frac{1}{(x+1)(x-1)}$

$\frac{x(x+1) - x(x+1)}{x^2 - 1} < \frac{1}{x^2 - 1}$

$\frac{\cancel{2x+1-2x-1}\ \ x^2 + x - x^2 + x}{x^2 - 1} < \frac{1}{x^2 - 1}$

$\frac{2x}{x^2 - 1} < \frac{1}{x^2 - 1}$