MA CV 29/10/14 (1)

CV 3 / 1 / 2

$a_n \geq 0$

$a = \lim_{n \to \infty} a_n$

DOKAŽTE:

$\lim_{n \to \infty} \sqrt{a_n} = \sqrt{a_n}$

$\forall \varepsilon > 0\ \exists N \in \mathbb{N} : \forall n > N : |\sqrt{a_n} - \sqrt{a}| < \varepsilon$

$a \neq 0$:

$$\frac{|\sqrt{a_n} - \sqrt{a}| \cdot |\sqrt{a_n} + \sqrt{a}|}{\sqrt{a_n} - \sqrt{a}} = \frac{|a_n - a|}{\sqrt{a_n} + \sqrt{a}} \leq \frac{|a_n - a|}{\sqrt{a}} \leq$$

$$\leq \frac{\varepsilon'}{\sqrt{a}} = \varepsilon \qquad \varepsilon' = \sqrt{a} \cdot \varepsilon$$

$a_n = 0$:

$a_n < \varepsilon' \qquad \varepsilon' := \varepsilon^2$

$a_n < \varepsilon$

CV 3/4

$$a_n = \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{3}\cdots\frac{2n}{2n-1}\cdot\frac{2n}{2n+1}$$

1) $a_n \to a$

2) $a \neq 0$

1) $$\frac{2n\,2n}{(2n-1)(2n+1)} < \frac{(2n+2)(2n+2)}{(2n+1)(2n+3)}\cdot\frac{2n\,2n}{(2n-1)(2n+1)}$$

$$\frac{4n^2}{4n^2-1} > 1$$

$$\cancel{\frac{2n}{2n-1}\cdot\frac{2n}{2}}$$

$$\underbrace{\frac{2}{1}}_{2}\cdot\underbrace{\frac{2}{3}}_{<1}\cdots\underbrace{\frac{2n}{2n-1}}_{a<1}\cdot\underbrace{\frac{2n}{2n+1}}_{<1} < 2$$

2) SPOREČ

$\forall \varepsilon > 0\ \exists N \in \mathbb{N} : \forall n \in > N : |a_n - 0| < \varepsilon$

ZVOLIME $\varepsilon = \frac{1}{2}$

$$|a_n| < \frac{1}{2}$$

$$a_n > 1 \Rightarrow |a_n| < \frac{1}{2} \quad \lightning$$

~~CU 3/7~~

CU 4/1/a

$$\lim_{n\to\infty} \frac{2n^2+n-3}{n^3-1} = \frac{\frac{2n^2}{n^3}+\frac{n}{n^3}-\frac{3}{n^3}}{\frac{n^3}{n^3}-\frac{1}{n^3}} =$$

$$\frac{0+0-0}{1-0} = 0$$

CU 4/1/c

$$\lim_{n\to\infty} \frac{2n^5+3n^3-2}{n^5-3n^3+1} = \lim_{n\to\infty} \frac{\frac{2n^5}{n^5}+\frac{3n}{n^5}+\frac{2}{n^5}}{\frac{n^5}{n^5}-\frac{3n^3}{n^5}+\frac{1}{n^5}} =$$

$$= \frac{2+0+0}{1+0} = 2$$

CU 4/1/E

$$\lim_{n\to\infty} \frac{(2^n+n^2)^2}{(5+\frac{1}{n})^n} = \lim_{n\to\infty} \frac{4^n+2^n\cdot 2n^2+n^4}{(5+\frac{1}{n})^n} =$$

$$\lim_{n\to\infty} = \frac{\frac{4^n}{5^n}+\frac{2^n\cdot 2n^2}{5^n}+\frac{n^4}{5^n}}{\frac{(5+\frac{1}{n})^n}{5^n}} = \frac{0+0+0}{\sqrt[5]{e}} = 0$$

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CV 4/2/D

$$\lim_{n\to\infty}\left(\sqrt{n+\sqrt{n}}-\sqrt{n-\sqrt{n}}\right)=$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt{n+\sqrt{n}}-\sqrt{n-\sqrt{n}}\right)\left(\sqrt{n+\sqrt{n}}+\sqrt{n-\sqrt{n}}\right)}{\left(\sqrt{n+\sqrt{n}}\;\cancel{}-\sqrt{n-\sqrt{n}}\right)}$$

$$=\lim_{n\to\infty}\frac{n+\sqrt{n}-n+\sqrt{n}}{\sqrt{n+\sqrt{n}}+\sqrt{n-\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt{n}}{\sqrt{n+\sqrt{n}}+\sqrt{n-}}$$

$$=\lim_{n\to\infty}\frac{2}{\frac{\sqrt{n-\sqrt{n}}}{\sqrt{n}}+\frac{\sqrt{n-\sqrt{n}}}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2}{\cancel{\frac{\sqrt{n+\sqrt{n}}}{n}}\;\sqrt{1+\frac{\sqrt{n}}{n}}+\sqrt{\cancel{n}-\frac{\sqrt{n}}{n}}}$$

$$=1$$

CV 4/3/B

$$\lim_{n\to\infty} n^2\left[\left(1+\frac{3}{n}\right)^5+\left(1+\frac{5}{n}\right)^3\right]=$$

$$\lim_{n\to\infty} n^2\left(1+5\frac{3}{n}+\binom{5}{2}\left(\frac{3}{n}\right)^2+5\left(\frac{3}{n}\right)^4+\left(\frac{3}{n}\right)^5-\right.$$

$$\left.-\left(1+3\frac{5}{n}+3\left(\frac{5}{n}\right)^2+\left(\frac{5}{n}\right)^3\right)\right)=\lim_{n\to\infty}\left(\frac{90+90}{n}+\frac{5\cdot 3^4}{n^2}+\right.$$

$$\left.+\frac{3^5}{n^3}-3\cdot 5^2-\frac{5^3}{n}\right)=15$$

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CV 4/3/E

$$\lim_{n\to\infty} \sqrt[n]{3^n + 5^n + 7^n} =$$

$$= \lim_{n\to\infty} 7 \cdot \sqrt[n]{\frac{3^n}{7^n} + \frac{5^n}{7^n} + 1} =$$

$$= \lim_{n\to\infty} 7\sqrt{1} = 7$$