5/1/A

$a_n \in O(c_n)$, $b_n \in O(d_n)$

$\overset{?}{\Rightarrow} a_n + b_n \in O(c_n + d_n)$

$\exists N_a \in \mathbb{N} : \exists C_a > 0 : \forall n \geq N : a_n \leq C_a \cdot c_n$

$\exists N_B \in \mathbb{N} : \exists C_b > 0 : \forall n \geq N : b_n \leq C_b \cdot c_n$

$C_x = C_a + C_b$

$n \geq \max(N_a, N_B) : a_n + b_n \leq C_a \cdot c_n + C_b \cdot d_n$

PLATÍ $\leq C_x(c_n + d_n)$

5/1/C

$\Rightarrow (a_n + b_n)^2 \in O(c_n^2) + O(d_n^2)$

$= O(c^2 + d^2)$

$a_n^2 + 2a_n a_n + b_n^2 \overset{?}{\leq} c(c_n + d_n)$

$a_n^2 \leq C_a^2 \cdot c_n^2$ $\boxed{2c_n d_n \leq c_n^2 + d_n^2}$

$b_n^2 \leq C_b^2 \cdot d_n^2$

$a_n^2 + b_n^2 \leq C(c_n^2 + d_n^2)$ ... $C = \max(C_a^2, C_b^2)$

$a_n b_n < C_a C_b c_n \cdot d_n < C_a C_b \left(\frac{c_n^2 + d_n^2}{2}\right)$

pozn: $2c_n d_n \leq c_n^2 + d_n^2$

$C = C_a + C_a \cdot C_B$

MA CV 19/11/14 2

$$\sqrt{xy} \le \frac{x+y}{2} \le \sqrt{\frac{x^2+y^2}{n}}$$

$$\sqrt{a_1 \dots a_n} \le \frac{a_1 + a_2 + \dots + a_n}{n} < \sqrt{\frac{a_1 + \dots + a_n}{n}}$$

$\lfloor x \rfloor$ ... DOLNÍ CELÁ ČÁST

$\lfloor 2{,}7 \rfloor = 2$; $\lfloor \sqrt{15} \rfloor = 3$; $\lfloor \sqrt{16} \rfloor = 4$

5/2/a DOKAŽTE NEBO VYVRAŤTE

$$\left(\frac{n^2}{\log \log n}\right)^{1/2} \in O\left(\lfloor \sqrt{n} \rfloor^2\right)$$

$\lfloor x \rfloor \le x$

$x - 1 \le \lfloor x \rfloor$

$\Downarrow$ PROTOŽE $x - 1 \le \lfloor x \rfloor$

$$\left(\frac{n^2}{\log \log n}\right)^{1/2} \in O\left(\sqrt{n-1}^2\right)$$

$n = n + 2\sqrt{n} + 1$

$\frac{n}{2} = n - 2\sqrt{n}$

$2\sqrt{n} \le n/2$

$4n \le n^2/4$

$16 \le n$

$$\frac{n}{\sqrt{\log \log n}} \in O\left(\frac{n}{4}\right)$$

$$c = 2\sqrt{\log \log (16)}$$

$\square$

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5/3/A

$$\sum_{n=1}^{\infty}\left(\frac{2}{2^n}+\frac{(-1)^n}{3^n}\right)$$

$6+\frac{3}{4}-(3-1)=2+\frac{3}{4}$

MUSÍME ODEČÍST 0-TÝ ČLEN, VZORECEK

$$S_n = a_0 \cdot \frac{q^n-1}{q-1}$$

JE OD 0

$$S_n = a_0 \cdot \frac{q^n-1}{q-1}$$

PRO $n \to \infty$:

$$S_\infty = a_0 \cdot \frac{1}{1-q}$$

$$a_n \to 3 \cdot \frac{1}{1-\frac{1}{2}} = 6$$

$$b_n \to \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$$

5/4/C KONVERGUJE? NE

$$\sum_{n=2}^{\infty} \frac{n^n}{(n+1)^n - 2n^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$ DIVERGUJE ~~KONVERGUJE~~

(HARMONICKÁ ŘADA)

$$\frac{1}{n} < \frac{n^n}{(n+1)^n - 2n^n}$$

$$\Updownarrow$$

$$1 < \frac{n^{n+1}}{(n+1) - 2n^n}$$

⋮

⋮

$$n \geq \left(\frac{n+1}{n}\right)^n$$

$\to \infty$ $\qquad \to e$

$$\Downarrow$$

$$\sum_{n=2}^{\infty} \frac{n^n}{(n+1)^n - 2n^n}$$

DIVERGUJE

□

5/4/D

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{(2n-1)(2n+1)}}$$

ZVOLÍME MENŠÍ DIVERGENTNÍ ŘADU

$$\sum_{n=1}^{\infty} \frac{1}{n} \quad , \quad \sum_{n=1}^{\infty} \frac{1}{4n}$$

$$\frac{1}{\sqrt{(2n-1)(2n+1)}} \geq \frac{1}{4n}$$

$$\frac{1}{(2n-1)(2n+1)} \geq \frac{1}{(4n)^2}$$

$$\frac{1}{(4n-1)^2} \geq \frac{1}{16n^2}$$

$$16n^2 \geq 4(4n^2-1)$$

PLATÍ

5/5/B

POKUD $\sum_{n=1}^{\infty} x_n$ KONVERGUJE A $x_n > 0$ TAK KONVERGUJE I $\sum_{n=1}^{\infty} x_n^2$

- $\lim_{n\to\infty} x_n = 0$
- $\exists N \in \mathbb{N} : \forall n \geq N : x_n \leq 1$
- $\exists N \in \mathbb{N} : n \geq N \Rightarrow x_n^2 \leq x_n$

$\lim_{n\to\infty} x_n = 0$

- x_n OMEZENÁ
- $x_n \leq C_n, n \in \mathbb{N}$
- $x_n^2 \leq x_n \cdot x_n \cdot C, n \in \mathbb{N}$

5/4/C

POKUD KONVERGUJE $\sum_{n=1}^{\infty} x_n$ A $x_n \geq 0$, PAK

KONVERGUJE I $\sum_{n=1}^{\infty} \frac{\sqrt{x_n}}{n}$.

$$\cancel{\frac{\sqrt{x_n}}{n} \leq x_n}$$

$$\cancel{\frac{1}{n} \leq \sqrt{x_n}}$$

$$\frac{\sqrt{x_n}}{n} = \sqrt{x_n + \frac{1}{n^2}}$$

$$\sqrt{x_n + \frac{1}{n^2}} \leq \frac{x_n + \frac{1}{n^2}}{2}$$

$$\sum_{n=1}^{\infty} \frac{x_n + \frac{1}{n^2}}{2} = \frac{\sum_{n=1}^{\infty} x_n}{2} + \frac{\sum_{n=1}^{\infty} \frac{1}{n^2}}{2}$$

5/4/E

KONVERGUJE?

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$$

$$\sqrt[n]{n} \to 1$$

ŘADA DIVERGUJE.