MATEMATICKÁ INDUKCE

$$\sum_{i=0}^{n} 2^{i} = 2^{n+1} - 1$$

$i \to i+1$

$i \to n+1$

$$= \sum_{i=0}^{n+1} = \left(\sum_{i=0}^{n} 2^{i}\right) + 2^{n+1}$$

$$= \sum_{i=0}^{n+1} 2^{n+1} - 1 + 2^{n+1}$$

$$= \sum_{i=0}^{n+1} 2^{n+2} - 1 = \sum_{i=0}^{n+1} 2^{(n+1)+1} - 1$$

$\square$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n+1} \left(\sum_{i=0}^{n} i\right) + n+1$$

$$\sum_{i=0}^{n+1} \frac{n(n+1)}{2} + n+1$$

$$\sum_{i=0}^{n+1} \frac{(n+1)(n+2)}{2}$$

$\square$

$$\sum_{i=0}^{n} i\,2^{i} = (n-1)\,2^{n+1} + 2$$

$$\sum_{i=1}^{n+1} \left(\sum_{i=1}^{n} i\,2^{i}\right) + (n+1)\,2^{n+1}$$

$$\sum_{i=1}^{n+1} (n-1)\,2^{n+1} + 2 + (n+1)\,2^{n+1}$$

$$\sum_{i=1}^{n+1} n\,2^{n+2} + 2$$

$\square$