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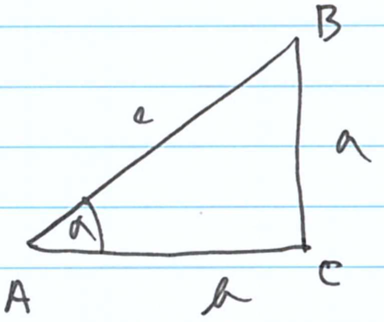
TRIGONOMETRIE

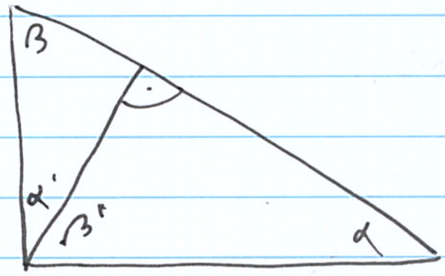
$\alpha \in \left\langle 0; \frac{\pi}{2} \right)$



$\cos \alpha = \frac{b}{c}$ $\quad$ $tg\, \alpha = \frac{a}{b}$

$\sin \alpha = \frac{a}{c}$ $\quad$ $cotg\, \alpha = \frac{b}{a}$

$$\boxed{\cos^2 \alpha + \sin^2 \alpha = 1}$$



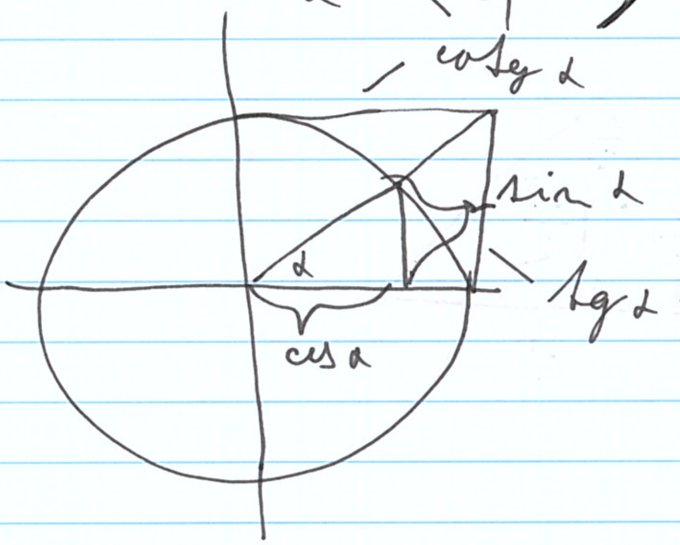
$$\frac{b^2}{c^2} + \frac{a^2}{c^2} = c^2$$

$$a^2 + b^2 = c^2$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

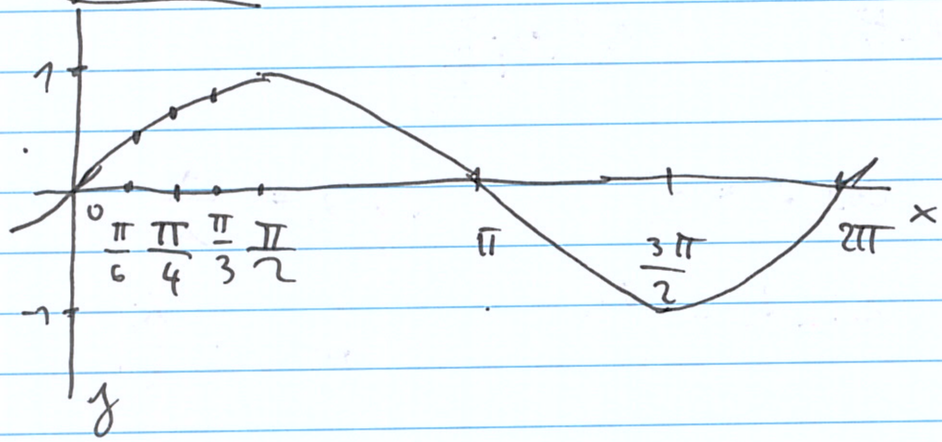
$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$\alpha \in \langle 0, 2\pi \rangle$

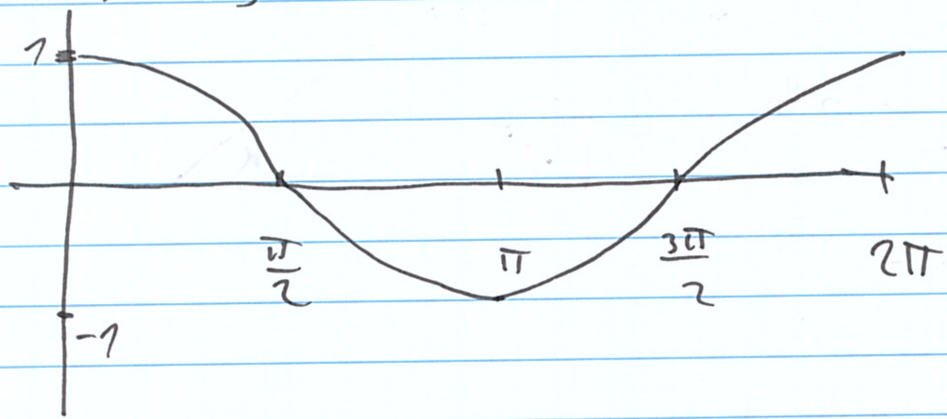


$$\sin(\pi - \alpha) = \sin \alpha$$
$$\cos(\pi - \alpha) = -\cos \alpha$$

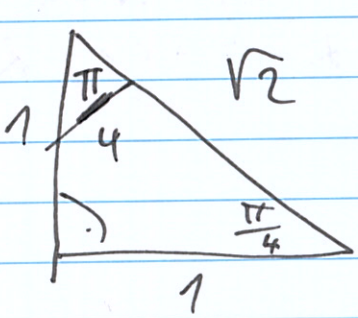
Sinus

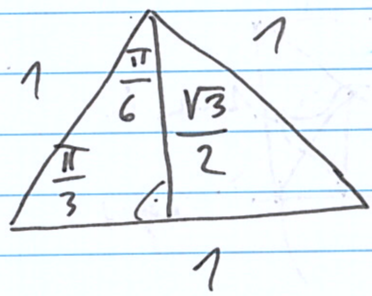


Cosinus



α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\alpha)$	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	1
$\cos \alpha$	1	$1/2$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—





RCE

$$\cos \alpha = -\frac{\sqrt{3}}{2}$$

$$\alpha = \frac{5\pi}{6} + 2k\pi \qquad k \in \mathbb{Z}$$

$$\alpha = \frac{7\pi}{6} + 2k\pi$$

$$2 \cdot \sin\left(3 - \frac{\pi}{2}\right) = \sqrt{3}$$

$$y = 3x - \frac{\pi}{2}$$

$$\sin y = \frac{\sqrt{3}}{2}$$

$$y = \frac{\pi}{3} + 2k\pi \qquad k \in \mathbb{Z}$$

$$y' = \frac{2\pi}{3} + 2k\pi$$

$$x = \frac{y}{3} + \frac{\pi}{6}$$

$$x = \left(\frac{\pi}{9} + \frac{\pi}{6}\right) + \frac{2}{3}k\pi$$

$$= \left(\frac{2\pi}{9} + \frac{\pi}{6}\right) + \frac{2}{3}k\pi$$

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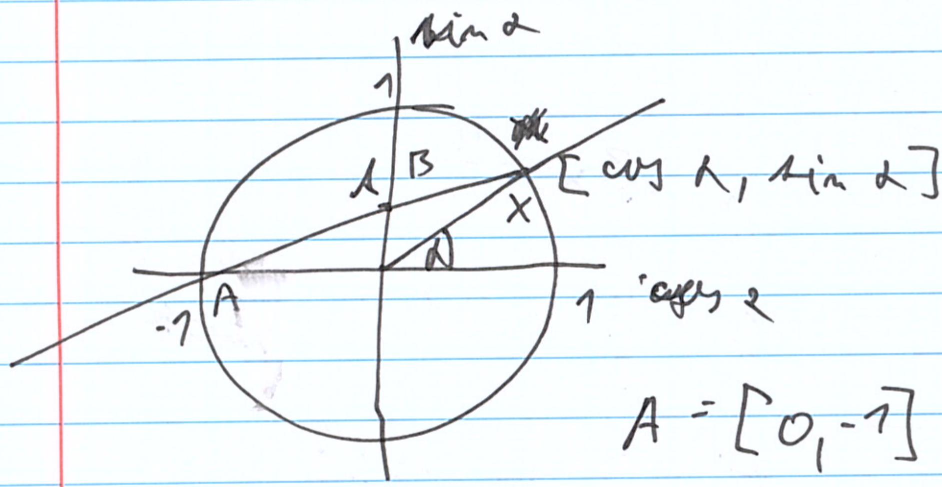
$$\mathrm{tg}^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \dots$$

$$\boxed{\cos^2\alpha = \frac{1}{1+\mathrm{tg}^2\alpha}}$$

$$\sin^2\alpha = \frac{\mathrm{tg}^2\alpha}{1+\mathrm{tg}^2\alpha}$$



$$t = \mathrm{tg}\frac{\alpha}{2}$$

$$A = [0, -1]$$

$$B = [0, t]$$

$p_1 : [-1, 0] + s(1, t)$

$x = -1 + s$

$y = st$

$$(-1+s)^2 + (st)^2 = 1$$

$\vdots$

$$s(s + st^2 - 2) = 0 \qquad s \neq 0$$

$$\boxed{x = \frac{1-t^2}{1+t^2}}$$

$$y = st = \frac{2t}{1+t^2}$$

$$\boxed{X = \left[\frac{1-t^2}{1+t^2} ; \frac{2t}{1+t^2}\right]}$$

24/9/14

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

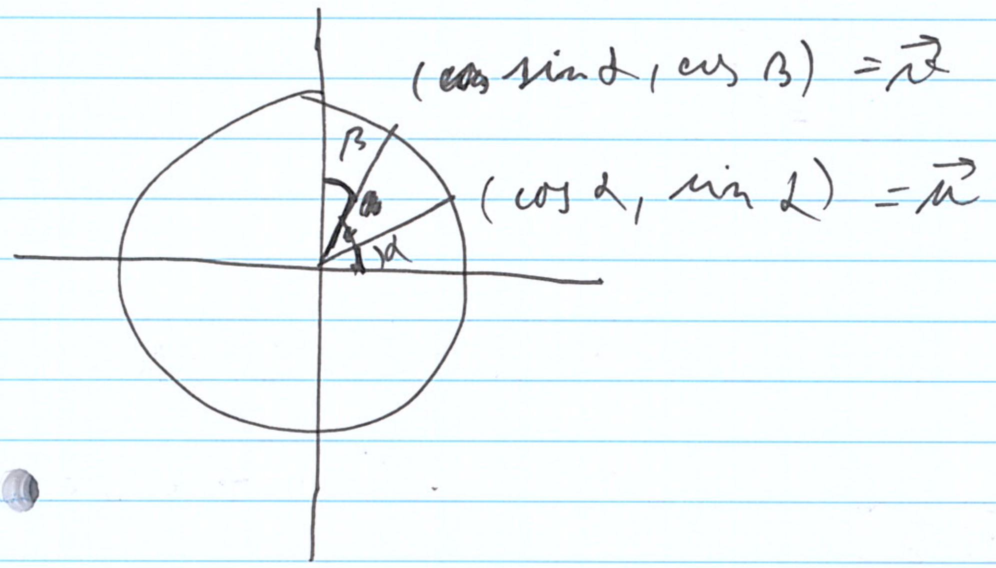
$$\sin(x-y) = \sin x \cos y - \cancel{\sin} \cos x \sin y$$

$$\cos(x+y) = \sin\left(\frac{\pi}{2} - \cancel{y}\, x - y\right) =$$

$$= \sin\left(\frac{\pi}{2} - x\right)\cos y - \cos\left(\frac{\pi}{2} - x\right)\sin y =$$

$$= \cos x\, \cancel{\cos}\, \cos y - \sin x \sin y$$

$$\cancel{\text{sen}}\ \cos(x-y) = \cos x \cos y + \sin x \sin y$$



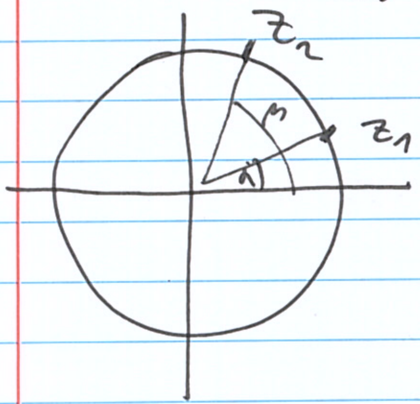
$$\frac{\vec{u}\ \vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \vec{u}\cdot\vec{v} = \cos\varphi = \cos\left(\frac{\pi}{2} - \alpha - \beta\right).$$

$$\cancel{\ } = \sin(\alpha + \cancel{\alpha}\,\beta)$$

$$\vec{u}\cdot\vec{v} = \cos\alpha \sin\beta + \sin\alpha\cos\beta$$

24/9/14

$\cos\alpha + i\sin\alpha$



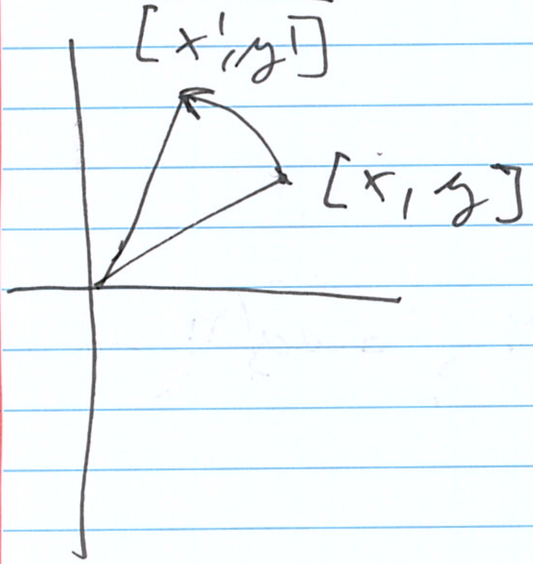
$\cos\alpha + i\sin\alpha$

~~zi~~

$$z_1 \cdot z_2 = (\cos\alpha\cdot\cos\beta - \sin\alpha\sin\beta) +$$
$$i(\cos\alpha\cdot\sin\beta + \sin\alpha\cdot\cos\beta) =$$
$$= \cos(\alpha+\beta) + i\sin(\alpha+\beta)$$

POSUNUTÍ	STEJNOLEHLOST
$\vec{v} = (1,2)$	
$x' = x + 1$	$x' = 3x$
$y' = y + 2$	$y' = 3y$

OTOČENÍ



$$x' = x\,\cancel{\sin}^{\cos} - y\sin\alpha$$

$$y' = x\,\cancel{\sin}\alpha + \cos\alpha$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x \\ \dots \end{pmatrix}$$

PŘ ~~(10)~~ A) 24/9/14 7

$$\sin x + \sqrt{3}\cos x = 1$$

$$\sqrt{3}\cos x = 1 - \sin 2x$$

$$3\cos^2 x = 1 - 2\sin x + \sin^2 x$$

~~3 (1-cos)~~

$$3(1-\sin^2 x) = 1 - 2\sin x + \sin^2 x$$

$$4\sin^2 x - 2\sin x - 1 = 0$$

SUBSTITUCE

$$\sin x = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

$$\boxed{x = \frac{\pi}{2} + 2k\pi} \qquad k \in \mathbb{Z}$$

~~$= \frac{3\pi}{2} + 2k\pi$~~

$$\boxed{\begin{aligned} x &= \frac{7\pi}{6} + 2k\pi \\ x &= \frac{11\pi}{6} + 2k\pi \end{aligned}}$$

ZKOUŠKA =>

$$x \in \left\{ \frac{\pi}{2} + 2k\pi ; \frac{11\pi}{6} + 2k \right.$$

B) $$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}$$

$$\cos\alpha \sin x + \sin\alpha \cos x = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3}$$

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$$

KDO NEUMÍ MLČET NEUMÍ ANI MLUVIT

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - \cancel{\text{1-cos}}\, 1 + \cos^2 = 2\cos^2 x - 1$$

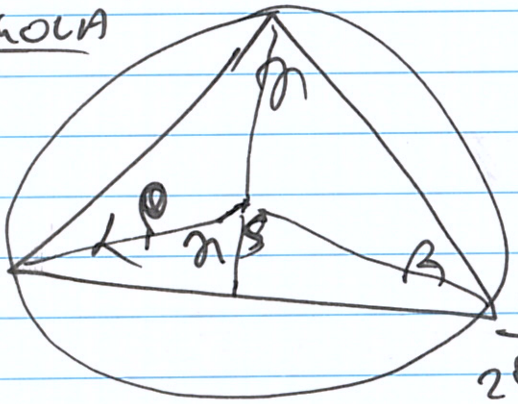
$$|\cos x| = \sqrt{\frac{1+\cos^2 x}{2}}$$

~~|cos~~

$$|\sin x| = \sqrt{\frac{1-\cos^2 x}{2}}$$

SINOVÁ A KOSINOVÁ VĚTA

SINOVÁ

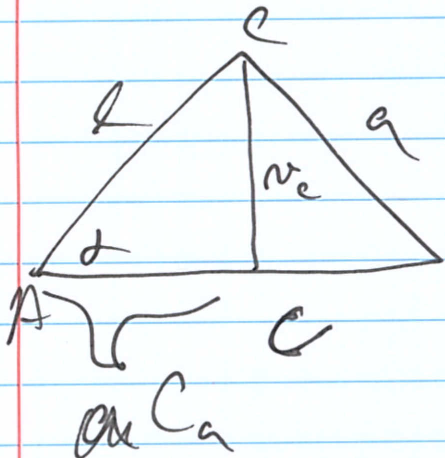


$$S = \frac{1}{2} b \cdot c \cdot \sin \alpha$$

$$\frac{1}{2\rho} = \frac{\sin \gamma}{c} = \frac{\sin \alpha}{a} = \frac{\sin}{b}$$

$$\sin \gamma \; \frac{\frac{c}{2}}{\rho} = \frac{1}{2\rho}$$

KOSINOVÁ



$$a^2 = b^2 + c^2 - 2bc\cos$$

$$v_c^2 = a^2 - (c - c_a)^2$$

~~$v_c^2 = \dots$~~

$$v_c = b^2 - c_a^2$$

ELEMENTÁRNÍ FUNKCE

24/9/2014

- POLYNOMY – 1, X
- GONIOMETRICKÉ FCE – sin
- EXPONENCIÁLY – e^x
- ~~LOMENÉ (RACIONÁLNÍ) FCE~~
- ~~LOGARITMICKÉ FCE~~

- OPERACE: +, −, *, /

INVERZNÍ – f^{-1} JE INVERZNÍ K f, POKUD

SKLÁDÁNÍ

$$f(f^{-1}(x)) = x = f^{-1}(f(x))$$

DEF. ELEMENTÁRNÍ FCE JSOU $1, x, e^x, \sin x$. A VŠECHNY DALŠÍ, KTERÉ Z NICH VZNIKNOU KONEČNÝM OPAKOVÁNÍM OPERACÍ +, −, *, /, SKLÁDÁNÍ A INVERZE.

EXPONENÍ FCE

$e = 2{,}718281828\ldots$ IRACIONÁLNÍ ČÍSLO

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e^0 = 1$$

$$e^{x+y} = e^x \cdot e^y$$

$$e^{-x} = \frac{1}{e^x}$$

$a \neq 0$ $y = a^x$

$a \neq 1$

$a \in \mathbb{R}(0; \infty)$

INVERZNÍ FCE e^x

$D(e^x) = \mathbb{R}$

$H(e^x) = (0; \infty)$

PROSTOTA:

$x \neq y;\ x, y \in D(e^x) \Rightarrow e^x \neq e^y$

„ln je inverzní k e^x"

$\ln(e^x) = x \qquad x \in \mathbb{R}$

$e^{\ln x} = x \qquad x \in (0;\infty)$

! $\forall x \in \mathbb{R}$: $\ln(e^x) \neq e^{\ln x}$ $\qquad$ $\forall x \in (0;\infty)$: $\ln(e^x) = e^{\ln x}$

LOGARITMUS

$\ln(x\,y) = \ln(x) + \ln(y)$

$\ln(1) = 0$

$\ln(x^n) = n\ln(x)$

OBECNÁ EXP. FCE

$a \neq \{0, 1\}$

$a \in \cancel{\mathbb{R}}\,(0;\infty)$

$a^x = y$

$e^{x \ln a} = y$

$\boxed{a^x = e^{x \ln a}}$

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OBECNÝ LOGARITMUS

$$\log_a x = y \overset{DEF}{\iff} a^y = x$$

$$a^{\log_a x} = x \quad \forall x \in (0;\infty)$$

$$\log_a x = \cancel{\log} \frac{\ln(x)}{\ln(a)}$$

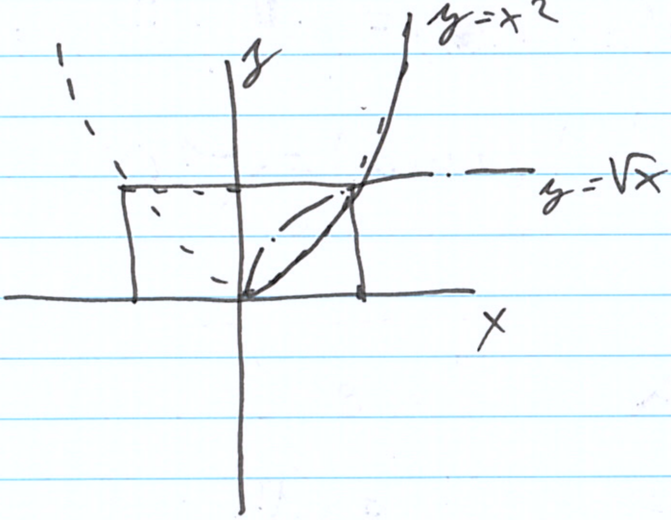
Př: $\log_{\frac{1}{3}} x = \dfrac{\ln(x)}{\ln(\frac{1}{3})} = \dfrac{-\ln x}{\ln 3} = -\ln_3 x$

ODMOCNINA

$$\left(x^2/\langle 0;\infty)\right)^{-1} =: \sqrt{x}$$

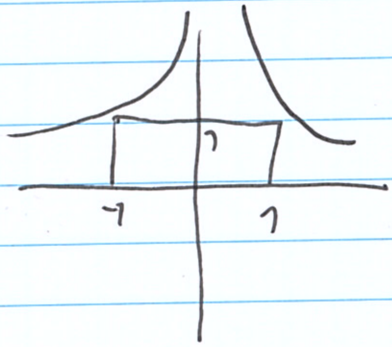
$$D(x^2_z) = \langle 0;\infty) = H(\sqrt{x})$$

$$H(x^2_z) = \langle 0;\infty) = D$$

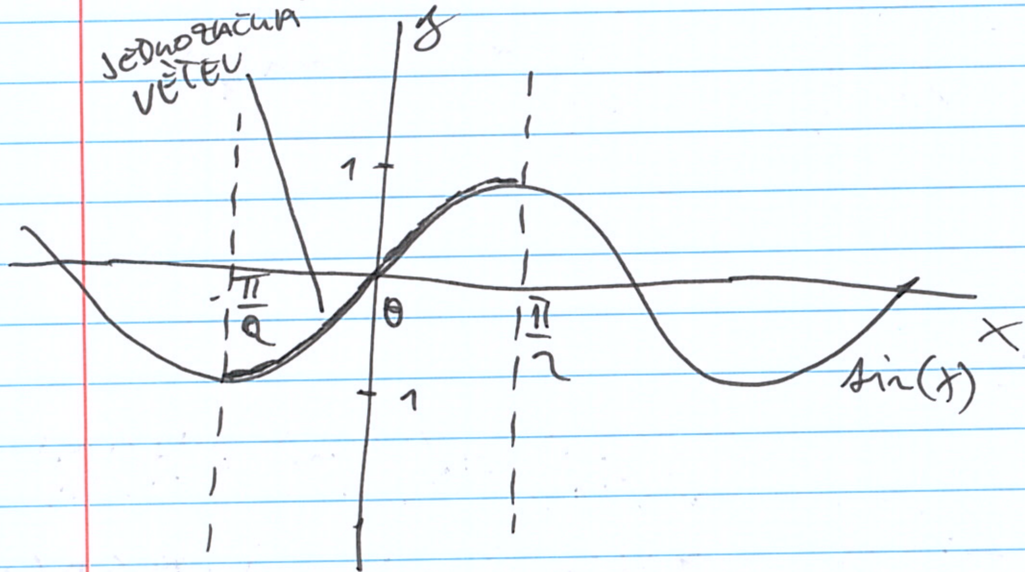


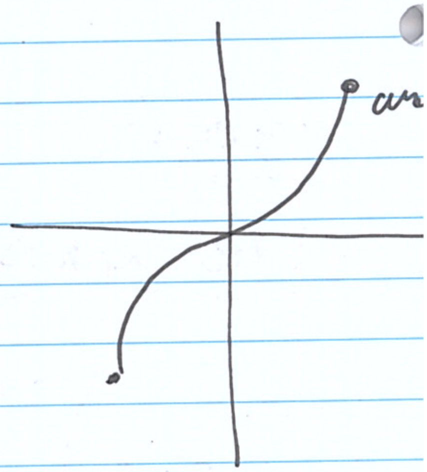
PŘEVRÁCENÉ HODNOTY SUDÝCH MOCNIN

$y = \frac{1}{x^2}$



INVERZE K SIN(x)





$\sin\Big|_{\langle -\frac{\pi}{2} ; \frac{\pi}{2} \rangle} =: \arcsin(x)$

$D(\text{ARCSIN}) = \langle -\frac{\pi}{2} ; \frac{\pi}{2} \rangle$

$H(\arcsin) = \langle -1 ; 1 \rangle$

$$\boxed{\begin{array}{ll} \sin(\arcsin(x)) = x & \forall x \in \langle -1, 1 \rangle \\ \arcsin(\sin(x)) = x & \forall x \in \langle -\frac{\pi}{2} ; \frac{\pi}{2} \rangle \\ \neq x & \end{array}}$$

$\forall x \in \mathbb{R} \setminus \langle -1 ; 1 \rangle$

⇒ INTERVALY, ~~TRÁSEM~~ VZÁJEMNÉ JEDNOZNAČNOSTI

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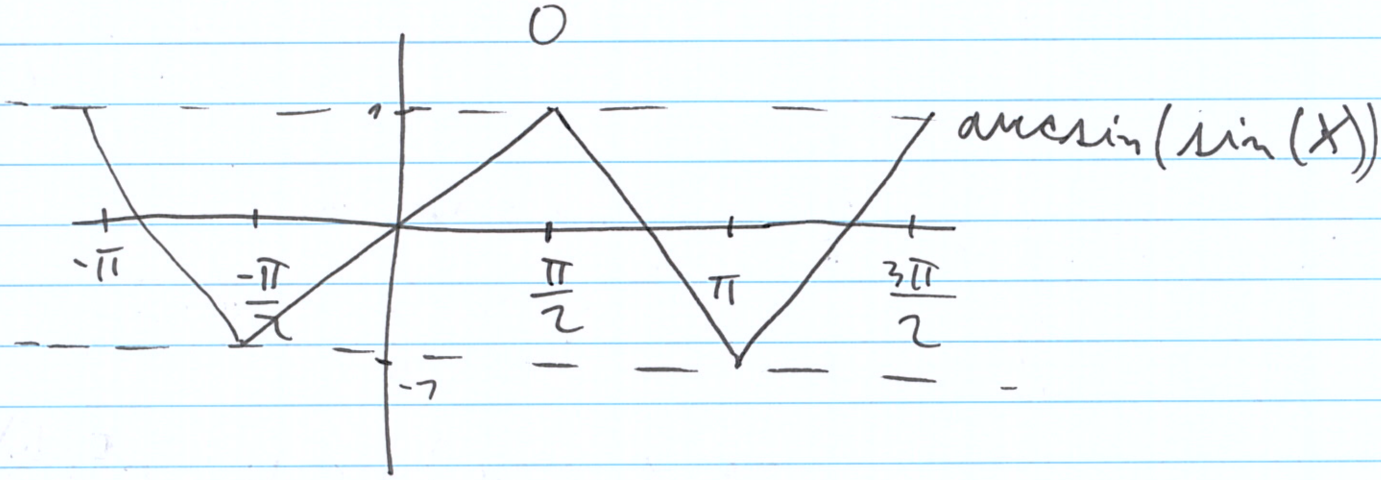
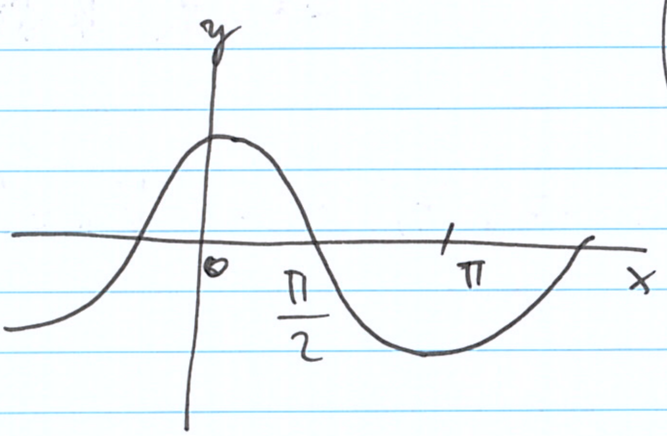
$$\arcsin(\underbrace{\sin\pi}_{0}) = 0$$



$(\cos x)^{-1}$?



$$\left(\cos x\big/_{\langle 0;\pi\rangle}\right)^{-1} =: \mathrm{arcc}$$

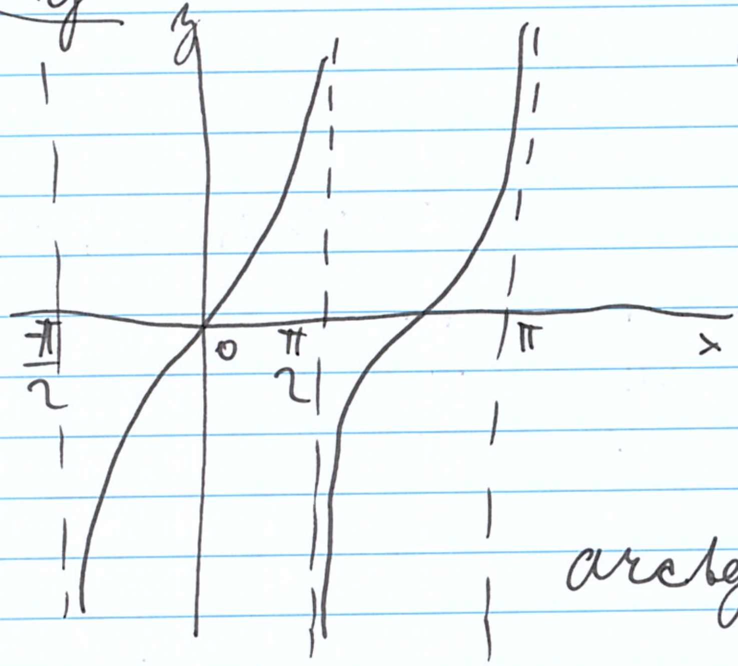
$$D(\arccos(x)) = \langle -1;1\rangle$$

$$H(\arccos(x)) = \langle 0;\pi\rangle$$

$$\boxed{\begin{array}{ll} \cos(\arccos(x)) = x & \forall x \in \langle -1;1\rangle \\ \arccos(\cos(x)) = x & \forall x \in \langle 0;\pi\rangle \end{array}}$$

$$\neq x \quad \forall x \in \mathbb{R} \setminus \langle -1;1\rangle$$

tg



$\text{arctg}\,x := \left(\text{tg}(x)\big|_{\left(-\frac{\pi}{2};\frac{\pi}{2}\right)}\right)$

$D(\text{arctg}) = \mathbb{R}$

$H(\text{arctg})\ \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$

$\text{arctg}(\text{tg}(x)) = x \quad \forall x \in \left(-\frac{\pi}{2};\right.$

$\neq x \quad \forall x \in \mathbb{R} \setminus \left(\frac{\pi}{2}\right.$

$\text{tg}(\text{arctg}(x)) = x \quad \forall x \in \mathbb{R}$